NCTM Research Catalyst Conference

At the Sheraton Reston Hotel,
Reston, VA

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I. Why is reform needed?

Because we are faced with important problems and challenges

Two categories:

A. Justification problems of societal significance

- **Imbalances in the qualifications of the work force:**

- insufficient enrolment to school programmes and studies with a substantial mathematical component. (Society demands people with qualifications from such programmes.)

- programmes with a mathematical component become de-mathematised in order to better attract students

- **The relevance paradox**

- discrepancy between the objective relevance of mathematics to society and its subjective irrelevance as perceived by many students

- **Motivation problems**

- maths is perceived as boring, meaningless, stereotyped, unchallenging, and too demanding, by many students

- **”Mathematics for all” is being questioned**

- by society at large, by some general educationists, and by some research mathematicians

mathematics education has problems at “delivering” (the TIMSS and PISA shocks in, say, Germany and Sweden)

ICT takes over

B. Problems within mathematics education

- **Ill-defined curricula**

- Muddy soup, in many countries, of goals and aims, content, activities, assessment

- Syllabusitis (mistake mastery for exposition to topics)

- **Problems of coherence**

- mathematics education means rather different things at different levels

- **Problems of transition**

- from primary to secondary (lower to upper), from secondary to tertiary education

- **Problems of progression**

- within as well as between levels

- **Problems of classroom heterogeneity across the same level**

- ”declaration of goods”

- level(s) of the target audience

- **Problems concerning differentiated teaching**

- what does it mean? and what’s the purpose of differentiated teaching?

  different ”treatment” to achieve the same result?

  same treatment allowed to lead to different results?

- lack of resources for differentiated teaching

- **Problems of assessment**

- discrepancy betweens aims and assessment modes

- difficulty of interpretation of outcomes

- **Problems with teachers' competencies**

- Enormous variation

- Primary and lower secondary: focus on method and pedagogy,
  insufficient insight into subject matter and didactics

- Upper secondary and tertiary: focus on subject matter,
  insufficient pedagogy and insight into didactics

II. This calls for rethinking of mathematics education

These problems and challenges **call for a thorough rethinking** of mathematics education and of the ways to describe it

When so doing, different targets are possible:

- Structure and organisation of the educational system, the school, the classroom
- **Curriculum**
- Teaching materials
- Ways of teaching
- Assessment modes and instruments
- **Education and professional development of teachers**

All of these are important, but here, particular focus on the **curriculum** and on **teachers**

What do we mean by "curriculum"?

And who is the curriculum authority?

compo-nent	**math. con-**	**"big ideas"**	**goals** &	T/L- **acti-**	**com-peten-**	**test-ing**

curr. authority	**tent**		**aims**	**vities**	**cies**	
Nat. govern-ment						
Loc. govern-ment						
School leader-ship						
Teacher (s)						

Third dimension: Degree of leeway for teachers

Look at which is perceived as the **key component:**

*** specific mathematical content,** (the classical notion of curriculum), **the syllabus**

i.e. a sequence of topics, notions, concepts, objects, methods, techniques, results

Sometimes and in some places:
syllabus = textbook (+ teachers' guide),
hence **curriculum = textbook**

May also include history or applications.

*** "big ideas"**,
i.e. content defined by means of **phenomenological categories** pertaining to the "real world" (space, space, chance, change, etc.)

*** goals and aims**
i.e. specifying **ultimate ends**, as well as **knowledge** or **skills** that students should possess.

*** teaching & learning activities**
i.e. the kinds of t/l activities that teachers should **orchestrate** and students gain **experience** with and from (e.g. exercises, problem solving, modelling, explorative, data collection activities)

*** mathematical competencies**
i.e. the competencies which students should possess as a result of schooling

*** testing**

i.e. form and content of the ways in which students are tested at crucial points in time, e.g. at the end of school

In the ***NCTM Principles and Standards*****:**

A mixture of content strands and competencies:

Content strands associated with specific competencies:

- Number and operations
- Algebra
- Geometry
- Measurement
- Data analysis and probability

Competencies, independent of content strands:

- Problem solving
- Reasoning and proof
- Communication
- Connections

- Representation

III. The Danish KOM project

”One should ask whose knowledge is best, not who knows the most”

(Montaigne ”On Pedagogy”, Essays, 1st Book, Chapter 25)

More specifically, we should ask

“What does it mean to master mathematics?”

Our answer, in the **Danish KOM project:**

A competency based description of mathematical education

Definition:
To possess **mathematical competence** means to have knowledge of, to understand, to do, and to use mathematics, and to have a well-founded opinion about

it, in a variety of contexts **where mathematics plays or may play a role**.

***A* mathematical competency** is a distinct major constituent in mathematical competence.

Two groups of competencies

The ability to ask and answer question in and with mathematics:

Mathematical thinking competency – mastering mathematical modes of thought

- understand and deal with the roots, scopes, and limitations of given concepts
- abstract concepts, generalise results
- distinguish between different types of mathematical statements

- awareness of questions typical of maths, and insight into the types of answers to be expected

- ability to pose such questions

Ex: ”Is it true that any perimeter, however large, can be attained amongst the rectangles of a given area? Yes! Is the dual statement true? No, the maximal area is attained by the square having the given perimeter.”

Problem handling competency – formulating and solving mathematical problems

- detect, formulate, delimitate, and specify mathematical problems, pure or applied, open or closed

- ability to solve problems, posed by oneself or by others, if desirable in different ways

Ex. If we had only coins of values 3 and 5, which amounts could we pay with those coins?

Modelling competency – being able to analyse and build mathematical models concerning other areas

- analyse the foundations and properties of existing models, and assess their range and validity

- perform active modelling in given contexts
  i.e. structure and mathematise situations, handle the resulting model, drawing mathematical conclusions from it, validate the model, analyse it critically, communicate about it, monitor and control the entire process

Ex. What's the cost of using a cell phone?

Reasoning competency – being able to reason mathematically

- follow and assess others' mathematical reasoning

- understanding what a proof is (not) and how it differs from other kinds of reasoning

- understanding the logic behind a counter example

- uncover the main ideas in a proof

- devise and carry out informal and formal arguments, thus transforming heuristic reasoning to valid proofs

Ex. Where is the mistake in the following "proof": As $x+1 = (x^2-1)/(x-1)$, and as $1^2-1 = 0$, we have by inserting $x = 1$ that $2 = 0$.

The ability to deal with mathematical language and tools:

Representation competency – being able to handle different representations of mathematical entities

- understand (decode, interpret, distinguish) and utilise different kinds of representations of mathematical entities

- understand the relations between different representations of the same entity

- choose and switch between different representations

Ex. Consider various representations of an affine function, say $f(x) = -3x + 8$.

Symbol and formalism competency – being able to handle symbol language and formal mathematical systems

- decode symbol and and formal language

- translate back and forth between symbol language and natural language

- treat and utilise symbolic statements and expressions, including formulae

- insight into the nature of formal mathematical systems

Ex. Solve the equation $x(y+z) = xy + z$ in $(x,y,z) \in \mathbf{R}^3$.

Communication competency – being able to communicate, in, with, and about mathematics

- study and interpret written, oral or visual mathematical expressions or texts

- express oneself in different ways, and with different levels of precision, on mathematical matters to different audiences

Ex. Explain why it is illegal to divide by 0.

Aids and tools competence – being able to make use of and relate to the aids and tools of mathematics

- having knowledge of the existence and properties of different relevant tools

- insight into the possibilities and limitations of such tools

- reflectively using tools and aids

Ex. Handle visualisation software. Know which kinds of numbers can(not) be represented in a pocket calculator.

The competencies

- are closely **related**, yet **distinct**

- have a **dual** nature

- comprise **intuition** and **creativity**

- are **specific to mathematics**, yet **overarching** across educational levels and topic areas, from K to PhD

Overview and judgement regarding mathematics as a discipline

Focus on mathematics as a whole, not on mathematical situations

- The **actual application** of mathematics in other subject and practice areas

- The **historical development** of mathematics, both internally and from a social point of view

- The **nature** of mathematics as a discipline

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How do we use this in maths education?

- **normatively**
- including monitoring coherence and progression

- **descriptively**
  - including comparison of teaching and curricula
  - including identification of the causes of transition problems

- **metacognitive support** for teachers and students

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Relations between competencies and subject matter

A given competence can (only) be

- **exercised** in dealing with mathematical subject matter

- **developed** in dealing with mathematical subject matter

Choice of curriculum topics does not follow from the focus on competencies

Competencies and mathematical disciplines/topic areas are **orthogonal**:

	Thinking Problem handling	...	Tools and aids
Topic 1			
Topic 2			
. . .			
Topic n			

Each cell: Specifies how the corresponding competency manifests itself when dealing with the area at issue, at a given educational level

Topics chosen for grade 1 to introductory university:

- The number domains
- Arithmetic
- Algebra
- Geometry
- Functions
- Real analysis
- Probability theory
- Statistics
- Discrete mathematics
- Optimisation

Not all topics are represented at all levels

Description in aggregated terms, not detailed lists of concepts, topics, methods

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Curricula in Denmark are in a process of reform along the lines just outlined – at all educational levels.

Local working groups of implementation are being established in many places.

Inspiration to Norway and Sweden in their forthcoming reform endeavours.

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“What does it mean to be a good mathematics teacher?”

KOM’s answer:

A good mathematics teacher is one who can foster the development of mathematical competencies with her/his students

This requires that the teacher possesses the mathematical competencies her/himself.

Moreover: **Didactical and pedagogical competencies with specific regard to mathematics:**

- **Curriculum competency**:

To analyse, assess, relate to, and implement existing mathematics curricula and syllabi, and to construct new ones

- **Teaching competency**:

To devise, plan, organise, orchestrate and carry out mathematics teaching, including: creating a rich spectrum of teaching/learning situations; find, assess, select and create teaching materials; inspire and motivate students; discuss curricula and justify teaching/learning activities with students.

- **Uncovering of learning competency**:

To uncover, interpret and analyse students' learning of mathematics, as well as their notions, beliefs and attitudes towards mathematics. Includes identifying development with the individual student

- **Assessment competency**:

To identify, assess, characterise, and communicate students' learning outcomes and competencies, so as to inform and assist the

individual student, and other relevant parties. Includes selecting, modifying, constructing, critically analysing, and implementing a varied set of assessment forms and instruments to serve different formative and summative purposes.

All of these**: for different categories of recipients in different situations, and at different levels, paying attention to the individual student's needs and opportunities**

- **Collaboration competency**:
To collaborate with different sorts of colleagues in and outside mathematics, as well as others (parents, authorities) concerning mathematics teaching and its conditions

- **Professional development competency**:
To develop one's own competency as a mathematics teacher (a meta-competency), including participate in and relate to activities of professional development, such as in-service courses, projects, conferences; reflect upon one's own teaching and needs for

development; keep oneself up-dated about new developments and trends in research and practice.

In **Denmark**, this represents a **major challenge** to pre-service education and in-service development for teachers of mathematics. In fact, it requires **fundamental reforms** of our systems to this end.

But **not a 0-1 problem**. Lot of room for local initiatives.

IV Research issues

The competency approach gives rise to many research questions, such as

- Which mathematical competencies **are actually being pursued in classrooms** in different places and at different levels?

- Which competencies are **needed, desirable, utilised** (respectively) in other fields and in different vocations and professions?

- Which competencies are valued in formative and summative **assessment**?

- Which competencies are involved in **the tests** employed in different institutions or organisations?

- Which competencies do different categories of **mathematics teachers** actually possess?

- How to **assess the competencies** in valid and reliable ways?

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- 10^{23} more questions